(Monotonic Sequence)

(Defn) Monotonic increasing sequence:- If $u_{n+1} \geq u_n$ for every value of n, then sequence $\{u_n\}$ is known as monotonic increasing in strict sense.

Example:- (i) Sequence 1, 1, 3, 3, 5, 5 ------ is monotonic increasing sequence.

(ii) 1, 3, 5 ----- $(2n-1)$ is monotonic increasing sequence in strict sense.

(Defn) Monotonic decreasing sequence:- If $u_{n+1} \leq u_n$ for every value of n, then the sequence $\{u_n\}$ is known as monotonic decreasing sequence.

Example:- $-1, -1, -3, -3, -5, -5,$ -------- is monotonic decreasing sequence.

If $u_{n+1} < u_n$ for every value of n, then the sequence $\{u_n\}$ is known as monotonic decreasing sequence in strict sense.

Example:- $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4},$ ------ $\frac{1}{n}$ --- are examples of m.

// Define convergent and divergent sequence, its examples.

Convergent Sequence:- A sequence which converges to some no. l is said to be convergent sequence.

Example:- (i) A constt. sequence $(a, a, a$ ----$)$ of real no. converges to the real no. a.

(ii) The sequence (a_n) where $a_n = \frac{1}{n}$ converges to the real no.(zero).

Divergent Sequence:- A Sequence is divergent if it is not Convergent i.e. it it does not have a limit.

Example:- ① The Sequence $\{a_n\}$ when, $a_n = n$ $(n = 1, 2, 3 - - -)$ is divergent.

② The Sequence $\{a_n\}$, when $a_n = \log(\frac{1}{n})$ diverges to $-\infty$.

QNo- A Convergent Sequence has a unique limit.

or

Prove that a Convergent Sequence determines its limit uniquely.

Proof:- Let $\{a_n\}$ be a Convergent Sequence and it Converges to l and also to l', where $l \neq l'$.

Now, we have to show that it has got a unique limit.

To Prove it, Put

$$\epsilon = \frac{1}{2}|l - l'|.$$

As $\{a_n\}$ Converges to l. then there exist a no. p such that,

$$|a_n - l| < \epsilon \text{ for all } n \geq p.$$

Again, Since $\{a_n\}$ also Converges to l' then there exist a natural no. q Such that,

$$|a_n - l'| < \epsilon \text{ for all } n \geq q.$$

Put $r = \max\{p, q\}$ Then,

$|l-l'| = |l-a_r+a_r-l'|$

$\leq |l-a_r| + |a_r-l'|$

$< \epsilon + \epsilon = 2\epsilon$

$= |l-l'|$

i.e. $|l-l'| < |l-l'|$

which is absurd.

Hence, $l = l'$.

Thus the limit is unique.

Q.No.- Prove that every Convergent Sequence is bounded.

Proof:- Let $\{S_n\}$ be a Convergent sequence and it has a limit.

Let $Lt.\ S_n = l$.

If $\epsilon = 1$ there exists $M \in N$ such that;

$|S_n - l| < 1 \ (n \geq M)$ — (a)

Now, $|S_n| = |S_n - l + l| \leq |S_n - l| + |l|$

$< 1 + |l|$ — (b)

If $K = max.\{|1|, |S_2| \text{———}, |S_{M-1}|\}$

then, $|S_n| < K + |l| + 1 \quad n \in N$.

Now, putting $K + |l| + 1 = K_1$.

then, $|S_n| < K_1$ for all $n \in N$.

i.e. the sequence $\{S_n\}$ is bounded.